THE CHINESE UNIVERSITY OF HONG KONG Department of Mathematics

MATH 2055 Tutorial 3 (Oct 5) $_{Ng Wing Kit}$

Definitions:

 $\lim_{x \to \infty} f(x) = L \text{ if and only if } \forall \epsilon > 0, \exists N, \text{ such that } \forall x > N, \text{ we have } |f(x) - L| < \epsilon$ $\lim_{x \to \infty} f(x) = +\infty (-\infty) \text{ if and only if } \forall M, \exists N, \text{ such that } \forall x > N, \text{ we have } f(x) > (<) M$

Find the following limits and prove it by definition.

1. $\lim_{x \to \infty} \frac{1}{x}$ 2. $\lim_{x \to \infty} \frac{1}{x^2 + 1}$ 3. $\lim_{x \to \infty} \frac{x^2}{x + 1}$ 4. $\lim_{x \to \infty} \left(\frac{1}{\frac{1}{x} + 1}\right)^2$

5.
$$\lim_{x \to \infty} (\sqrt{x+1} - \sqrt{x})$$